

Which chiral symmetry is restored in hot QCD?

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Abstract

We review the current status of the problem of chiral symmetry restoration, discussing an open question of whether at the $SU(N_f)_A$ restoration point $T = T_c$ the $U(1)_A$ chiral symmetry is or is not (approximately) restored. New lattice and instanton-based studies are considered, and some new calculations are suggested to clarify the issue. We also speculate on possible experimental manifestations of two possible scenarios.

1. The problem

For simplicity, we ignore all effects due to the non-zero quark masses, and consider QCD in the chiral limit, with N_f *massless* quarks. In this case the QCD Lagrangian is just a sum of two separate terms, including right- and left-handed quarks, which implies *two* chiral symmetries: $SU(N_f)_A$ and $U(1)_A$.

Their fate is well known to be different. The former one is spontaneously broken in the QCD vacuum but it is restored at high temperatures, above some critical point, denoted as $T = T_c$. Present lattice simulations with dynamical quarks suggest $T_c \approx 150 MeV$.

The $U(1)_A$ chiral symmetry is not related to Goldstone bosons, as Weinberg has first pointed out. It is now well known this is because this symmetry simply does not exist at quantum level, being violated by the 'chiral anomaly'. Its physical mechanism is also known [1], it is driven by the tunneling between the topologically different gauge vacua, described semiclassically by the instantons [2].

It is also known, that at high temperatures the instanton-induced amplitudes are suppressed due to the Debye-type screening [3, 4], and therefore (at some accuracy level) we expect this symmetry to be 'practically restored' at high T. Let us denote the point where it happens with some reasonable accuracy as $T_{U(1)}$.

The main question to be discussed below is the interrelation of the two temperatures, T_c and $T_{U(1)}$. Let us refer as 'scenario 1' to the case $T_c \ll T_{U(1)}$ in which the complete $U(N_f)_A$ chiral symmetry is restored only well inside the quark-gluon plasma domain. Another possible case¹ $T_c \approx T_{U(1)}$ is referred below as 'scenario 2': it implies significant changes in many hadronic channels around this phase transition point. As we will discuss below, these two scenarios lead to quite different predictions.

This important question is well known and it was already discussed in literature, but still we do not have a definite answer. In these comment, we are going to look again at its somewhat controversial history, and discuss what was recently done (and also can be done) in theory, numerical and 'real' experiments in order to answer it.

¹The case $T_c \gg T_{U(1)}$ does not seem to be possible.

Pisarski and Wilczek [5] have considered this question in connection with the order of the chiral phase transition. They have pointed out that in the special case $N_f = 2$ the 'scenario 1' is likely to lead to *the second order* transition. The reason is an effective Lagrangian describing the softest modes is essentially the Gell-Mann-Levy sigma model, same as for the $O(4)$ spin systems.

However, in that paper Pisarski and Wilczek have actually argued in favor of the 'scenario 2'. Their argument was as follows: 'if instantons themselves are the primary chiral-symmetry-breaking mechanism, then it is very difficult to imagine the unsuppressed $U(1)_A$ -breaking amplitude at T_c '. They have even mentioned that this amplitude should be at T_c at least an order of magnitude smaller than at $T=0$, although no details of this estimate were given.

As we will discuss below, both lines of reasoning have accumulated new evidences, so the question under consideration appears to be even more controversial, than it was a decade ago. We hope new studies, especially the lattice-based ones, can finally clarify it.

2. Is the $N_f = 2$ phase transition similar to that for $O(4)$ spin system?

During the last 2 years the large-scale numerical lattice simulations with dynamical fermions have addressed the issue of QCD phase transition, especially its order. It was indeed found that the case of 'many flavors' $N_f > 2$ (especially well studied case is 4) leads to sharp first order transition, with two phase coexistence etc., while in the particular case $N_f = 2$ such phenomena were not observed (see discussion and original references in [6] or proceedings of yearly lattice conferences).

Rajagopal and Wilczek [7] have argued that in this case one probably has a second order transition, analogous to that of the $O(4)$ spin system. They have therefore proposed a suitable effective Lagrangian, essentially a 3-dimensional σ model, and have applied it to dynamical simulations of the phase transition. Now, do the available lattice data really support these ideas?

The most straightforward way to test them is to compare the critical behaviour in both cases, testing whether the $N_f = 2$ QCD and the $O(4)$ spin system do or do not belong to the same universality class. (Similar comparison was done in the past for many

different phase transition, including 'deconfinement' one in $SU(2)_c$ pure gauge theory.)

The first critical index to compare is the one for the order parameter, for which the analogy [7] suggests

$$\langle \bar{\psi}\psi \rangle \sim |(T - T_c)/T_c|^{.38 \pm .01} \quad (1)$$

Unfortunately, it is not that simple to compare it to lattice data, because chiral order parameter is only obtained as a small-mass limit, and the masses used on the lattice cannot be made sufficiently small. Recent analysis [6] has concluded, that the data are consistent with $O(4)$ critical exponents, although say $O(2)$ ones are not also excluded.

The second obvious question is the behaviour of global thermodynamical quantities, such as the specific heat. The $O(4)$ spin system is believed to have a very amusing behaviour, with *positive* power²

$$C(T) \sim |(T - T_c)/T_c|^{.19 \pm .06} \quad (2)$$

It means that the singular contribution of the soft modes *vanishes* at the critical point, and in order to single it out the 3-ed derivative of the free energy should then be calculated.

However, lattice data for the $N_f = 2$ QCD actually do show a *huge peak* in the specific heat around T_c . It certainly implies, that many new degrees of freedom become available (or are significantly changed) in this region. What these degrees of freedom are, both in hadronic language and in the quark-gluon one, remains the major open problem in the field.

Of course, there is no logical contradiction here: apart of large (but smooth) peak one may eventually find a small 'kink', which is truly singular. And still, if one can get any hint from the available lattice data at all, I think they certainly indicate that point to the N_f QCD and the $O(4)$ spin system are very different physical objects, even as far as such global parameters as thermodynamical properties are considered.

3. Are both chiral symmetries restored simultaneously?

²As far as I know, it remains unknown whether the coefficient is positive or negative: thus one can have a dip or a peak.

We have already mentioned at the end of section 1 the arguments in favor of 'scenario 2' mentioned in [5]. Are there any new arguments for or against it?

The early conjuncture (see e.g.[8] and references therein) relating instantons with spontaneous breaking of the $SU(N_f)_A$ chiral symmetry has gained strong support during the last decade. From a qualitative idea one has come all the way to quantitative understanding of many details. It is now possible to calculate not only the quark condensate, but also such quantities as the pion or nucleon masses, using instanton-based model.

The latest development has resulted in quite detailed studies of the QCD correlation functions. The 'instanton liquid' model was shown to reproduce their behaviour for many mesonic and baryonic channels [9], in agreement with phenomenology [10] and lattice calculations [11]. Direct lattice studies [12, 13] have also shown direct connection between the quark condensate (and correlation functions) and instantons.

Thus, there is little doubt that instantons drive chiral symmetry breaking in the QCD vacuum. Unfortunately, all this development corresponds to $T=0$, while very little was done for the finite T case.

Although the Debye screening of large-size instantons [3] should strongly suppress them at high T , what exactly happens below T_c remains unknown. The so called 'Pisarski-Yaffe suppression factor' [4]

$$dn_{inst}(T) \sim dn_{inst}(T=0) \exp[-\pi^2 \rho^2 T^2 (\frac{2N_c}{3} + \frac{N_f}{3})] \quad (3)$$

can be applied *only* in the quark-gluon plasma phase³. Taking into account available lattice data on thermodynamics and Debye screening length, one can now argue that this relation is applicable above $T \approx 2T_c$. We also know now that the typical instanton radius of the 'instanton liquid' is $\rho \sim 1/3 fm$ [14]. Combining the two one finds instanton suppression by at least 2 orders of magnitude at $T \approx 2T_c$.

Now, is it possible that strong (e.g. by one order of magnitude, as in the original Pisarski-Wilczek estimates) *instanton suppression is already present at $T = T_c$* ? There

³ Whatever simple this argument is, it was not made in the original papers. For small size instantons 't Hooft determinant is calculated correctly even at $T=0$, but the temperature-dependent corrections to it are calculated under specific assumption that the heat bath is nothing but an ideal gas of quarks and gluons. Another way to see this is to note that the Debye screening is absent below T_c .

are several reasons to think it is not the case. Unfortunately, none of them is direct and sufficiently strong at the moment. Let me mention two of them.

The first are measurements of *T-dependence of the topological susceptibility* χ in pure gauge theories. Let me remind, that in absence of fermions, it is more or less direct measure of the instanton density. What is found [16] demonstrate that $\chi(T)$ remains constant (or even somewhat increases) through the deconfinement phase transition, while the high-T data are so far inconclusive. Note however, that deconfinement in pure gauge theory corresponds to relatively high temperatures, about 240 MeV, compared to chiral restoration we discuss in the rest of the paper.

The second argument is the following one. The pressure of quark-gluon plasma can be written as $p = p^{quarks,gluons}(T) - B(T)$, where the first term is perturbative black-body type contribution, and the second 'bag term' describes the *difference* between the non-perturbative energy at zero and non-zero T. Strong suppression of instantons at T_c implies that the second term is mostly the vacuum energy

$$B = b/128\pi^2 < (gG_{\mu\nu}^a)^2 > \approx 400 \text{ MeV}/fm^3 \quad (4)$$

(where b is the famous coefficient of the Gell-Mann-Low beta function). Comparing it with the perturbative term, one then finds that it is too small to get *positive pressure* of the plasma phase. More details on lattice data and 'melting' of quark condensate can be found in ref. [21], where it is concluded that instanton suppression factor at T_c is about 1/2 or so. Although these arguments are not of course strict, they do indicate that strong instanton suppression at T_c is very unlikely.

Moreover, the qualitative picture of instanton-driven chiral symmetry restoration has significantly changed since the days of original conjectures. In fact, *suppression* of instantons is not the only way to 'kill' the quark condensate. Not only the *number* of instantons is important, but also their *relative positions and orientations*. If the tunneling events, instantons and anti-instantons, are rearranged into some finite clusters with *zero topological charge*, such as well-formed 'instanton-anti-instanton molecules', all properties of the vacuum are completely changed. In particular, such ensemble leads to zero quark condensate, or restored chiral symmetry.

Numerical simulations of interacting 'instanton liquid' at finite T [15] shows that it is exactly this type of correlations which are rapidly build up in the critical region. 'Pairing' of instantons allow them in this case to interact all the way around the Matsubara torus. This is especially enhanced in case of many light quark flavors, since light quark do not propagate well into space direction at high temperatures.

To work out a detailed theory of such correlations will take some time, so a schematic model for this phenomenon was recently proposed and studied in ref. [17]. The instanton ensemble is assumed to be a *mixture* of two components; (i) a random 'liquid' and (ii) $I\bar{I}$ molecules ⁴. It is easy to generate such ensemble, for various values of the 'molecule fraction' $f_m = \frac{2N_{molecules}}{N_{all}}$, the main parameter of the model. Behaviour of the main order parameter, $\langle \bar{q}q \rangle$ on f_m is very similar as its T-dependence, measured on the lattice.

It was further found, that different correlation functions depend on it quite differently. Some correlators are rather insensitive to it. For example, the vector one does not change by more than about 10 % for all compositions, from $f_m = 0 - 1$. Even the pion correlator shows remarkable stability for $f_m = 0 - 0.8$, with subsequent rapid drop if its coupling constant toward $f_m = 1$, at which point it coincides with its scalar 'relatives' (see below). At the same time, some correlators show dramatic sensitivity to f_m , especially the scalar ones.

There is no place here for further details, let me now point out the main physics of the phenomenon. Its one side was repeatedly emphasized before (see e.g. [19]): as the quark condensate 'melts down', so does a 'constituent quark mass' (whatever it means) and hadrons in average should become lighter. However, there is another side of the story: the instanton-induced interaction between quarks 'melts' as well. In the channels where this interaction is *attractive*, such as ρ, π , the two effects work in the opposite directions and can cancel each other. However, in the channels where it is *repulsive*, such as axial and I=1 scalar ones, they work together and lead to much stronger effects.

4. What lattice observables can further clarify the problem?

⁴Note the difference with earlier treatment of the problem, e.g in ref. [18]: now these two components are not treated as two phases, for $T < T_c$ they are present simultaneously .

Let me start with the most obvious ones. For example, why not measure the T-dependence of the instanton density, either 'globally', by the topological susceptibility, or 'locally, by 'cooling' or by the fermion method?

We have already mentioned some results on T-dependence of the topological susceptibility in *pure gauge* theory: those are not yet very accurate, and can be much improved. The same question in theories with quark was studied by Bernard et al [20], but in this case $\chi(T)$ is simply proportional to the quark condensate, as Ward identities imply.

It is in principle possible to see 'molecular' contribution, substituting 4-volume by its surface in the definition of the topological susceptibility

$$\chi_{surface} = \lim_{surface \rightarrow \infty} \langle Q^2 \rangle \quad (5)$$

Unfortunately, other methods are not very sensitive to 'molecules'. 'Cooling' can lead to 'annihilation' of a correlated pair. 'Fermion' method can help only if the pair is not too close, so that they still lead to pretty small eigenvalues of the Dirac operator.

Another question, which was often asked: why lattice people do not measure directly the η' mass, which is obviously the one most intimately related with the $U(1)_A$ chiral symmetry and its (practical) restoration.

The answer⁵ is technical: in such case it is clear that one should calculate not only the so called 'one-loop diagram'

$$K_1(x, y) = - \langle Tr[\Gamma S(x, y) \Gamma S(y, x)] \rangle \quad (6)$$

but that of the 'two-loop' one

$$K_2(x, y) = \langle Tr[\Gamma S(x, x)] Tr[\Gamma S(y, y)] \rangle \quad (7)$$

as well, and the latter is much more difficult to evaluate because one has to invert the Dirac matrix many times.

However, one can come around this problem, by looking into the *scalar* correlators. Let us discuss for simplicity only two light flavors and use the old-fashioned

⁵It seems a strong argument, forbidding measurements of the η' mass: as for the correlator at not-so-large distances, I think the needed statistical accuracy may still be achieved.

notations, calling the isoscalar $I=0$ scalar channel a σ one, and isovector $I = 1$ scalar channel a δ one ⁶. Under $SU(2)_A$ transformations, σ is mixed with π , thus restoration of this symmetry at T_c require identical correlators for these two channels.

Another chiral multiplet is $\delta, \eta_{non-strange}$, where the last channel is the $SU(2)$ version of η' : at $T=0$ those are very heavy and are not considered in chiral Lagrangians, or course.

On the contrary, the $U(1)_A$ transformations mix e.g. π, δ type states, and thus its 'practical restoration' should imply that such type of correlators should become similar. Finally, if *both* chiral symmetries are restored, a simpler statement follows: left-handed quarks never become right-handed, therefore all $\pi, \eta_{non-strange}, \sigma, \delta$ correlators should become the same.

Now it is clear what one should do: *to measure the δ correlator*, for which one does not need the double-loop diagram ⁷, as a function of T . Comparison with the pion correlator will tell us where they become close enough, which defines the $T_{U(1)}$ under consideration.

Unfortunately, strong confusion was going on in lattice measurements of scalar correlators. A number of such measurements was reported [20], with Kogut-Susskind fermions. In this formulation different flavors live on different lattice sites, so the operator is quite different for σ, δ . Although σ operator (and σ label) was used, only the *one-loop* part $K_1^\sigma(x, y)$ was calculated. It was somehow implicitly assumed, that the two-loop one, which leads to separable contribution

$$K_2^\sigma(x, y) \rightarrow \langle \bar{q}q(x) \rangle \langle \bar{q}q(y) \rangle \quad (8)$$

at large distances $|x - y|$, is something like a constant, which should be subtracted.

However, there is no ground for such an assumption, and experience with σ correlator calculated in the 'instanton liquid model' [9] shows that. even after subtraction, there remains a significant contribution of some light states, presumably related to the

⁶Now particle data table denote notations f_0 and a_0 to $I=0,1$ scalars: however particular resonances listed there under these names hardly have anything to do with correlators under consideration.

⁷ For its charged component (e.g. $\bar{u}d$) it is trivial, because two quarks have different flavor: a one-line algebra shows why it is so for neutral component as well.

famous sigma peak in two pion scattering. As a result, σ correlator look quite different from what was found in these works.

Now come better news. The non-locality and difference between σ and δ operators are, after all, just lattice artifacts. In continuum limit, both one-loop diagrams become identical

$$K_1^\sigma(x, y) \approx K_1^\delta(x, y) \quad (9)$$

At $T=0$ we know that the δ channel does not have any prominent resonances and is dominated by multi-pion states with energy above 1 GeV. And indeed, such features are seen in δ correlator, calculated with Wilson fermions (e.g. [11]). Thus, we are able to test our conjuncture, find qualitative agreement. On this ground, let me speculate that it is also true at non-zero T , so results of [20] do *resemble δ rather than σ correlator*. If so, instead of the well anticipated $\pi - \sigma$ degeneracy at T_c they gave actually hint a toward more interesting statement, the $\pi - \delta$ degeneracy, which implies restoration of *both* chiral symmetries at T_c ('scenario 2').

Of course, further studies are needed to clarify this conjuncture. In doing this, it would be especially useful to repeat what was done by Gottlieb et al [20], namely measure separately the *difference* between scalar and pseudoscalar correlators, which has the most dramatic temperature (and quark mass) dependence.

Finally, another way of answering the question we discuss is to study *T -dependence* of the average values of certain four-fermion operators. Selecting those which are related to chiral symmetry breaking, one may get rid of perturbative contribution. However, the small mass extrapolation is still necessary, as for the quark condensate.

A quantity sensitive to $U(1)_a$ is well known, it is e.g. 't Hooft-type operator

$$O^{tHooft}(T) = \langle\langle [(\bar{u}_R u_L)(\bar{d}_R d_L)] \rangle\rangle \quad (10)$$

(where L,R stand for left and right components of the quark fields). Around or above T_c there is no quark condensate, and 'unpaired' instantons have very small density $O(m^{N_f})$ [1]. However, in the measurement of quantities like this one, the denominators of the quark propagators will take powers of quark masses back, recovering the famous 't Hooft

effective interaction. In a sense, the operator considered can induce a tunneling event by itself, which was not present in the vacuum without it. Unfortunately, it implies in practice that small fraction of configurations will produce a large signal: certainly not an easy way for measurements. However, this is what one should do, trying to understand the question under consideration.

Let us for completeness also mention some 'alternative $SU(N_f)$ order parameters', the expectation values of the following 4-fermion operators

$$O_1(T) = \langle\langle [\bar{u}_L \gamma_0 u_L - \bar{d}_L \gamma_0 d_L][L \rightarrow R] \rangle\rangle \quad (11)$$

$$O_2(T) = \langle\langle [\bar{u}_L \gamma_i u_L - \bar{d}_L \gamma_i d_L][L \rightarrow R] \rangle\rangle \quad (12)$$

$$O_3(T) = \langle\langle [\bar{u}_L \gamma_0 t^a u_L - \bar{d}_L \gamma_0 t^a d_L][L \rightarrow R] \rangle\rangle \quad (13)$$

$$O_4(T) = \langle\langle [\bar{u}_L \gamma_i t^a u_L - \bar{d}_L \gamma_i t^a d_L][L \rightarrow R] \rangle\rangle \quad (14)$$

As it is discussed in [22], those enter Weinberg-type sum rules at non-zero temperatures, related with the difference between vector and axial correlators.

5.Can one observe the difference between the two scenarios in 'real' experiments?

It is expected that in high energy heavy-ion collisions at RHIC and LHC, the system will spend a significant amount of time, of the order of 30 fm/c., in the so called 'mixed phase' at $T \approx T_c$. It is not easy to produce quark-gluon plasma, but, after it is produced, it also difficult to transform it into ordinary low-density matter.

A very fascinating possibility, suggested by Bjorken [23] and recently studied in [7] using 'quenched' scenario, is possible formation of large-amplitude classical pion field due to large critical fluctuations. In order to described it one should know which type of fields should be included in the effective Lagrangian, describing dynamics of the phase transition. Scenario 1 implies that it is a (3-d) variant of sigma model, while scenario 2 demand the inclusion of (at least) twice more soft modes.

I could not figure out any observable signals for both scalars, σ and δ even if fluctuations in their directions at the transition is equally strong to that of the pion. However, it is clear that scenario 2 implies *large fluctuation in $\eta_{non-strange}$ direction*, and those can in principle be observed.

First of all, as proven by the WA80 experiment, both π^0, η can be extracted in the $\gamma\gamma$ mode, even in large-multiplicity heavy-ion collisions with its huge combinatorial background ⁸.

Moreover, the η spectra should have clear signs of the $U(1)_A$ restoration, provided scenario 2 is the case. In this case η mesons are easily produced inside the fireball, being very light, but then, while trying to leave the system, they should then experience the influence of very strong collective potential. Its crude estimate is $V \sim (m_\eta - m_\pi) \sim 400 MeV$. In contrast to pions, for which it is much smaller, it is comparable to their kinetic energy at breakup $E_{kinetic} \approx 3T \sim 400 MeV$. As a result, a significant fraction of etas should be trapped inside the fireball, and eventually shifted to much lower kinetic energies. This phenomenon, analogous to electrons evaporated from the hot cathode, should lead to a peak in the eta spectrum at small p_t .

6. Summary

Summarizing our discussion, we can repeat that the main question discussed in these comments remain open.

At one hand, it looks plausible that 'phase transition in mathematical sense', the singularity, is related with 4 'soft' modes, pions and sigma ones. At another, it becomes increasingly clear that physics at $T \approx T_c$ cannot be described in terms of these fields alone, and excitations of matter in this region are very different from those at $T = 0$.

In particular, there are evidences that *parity partners* of pions and sigma, $\delta, \eta_{non-strange}$, which are very heavy or even non-existing at $T=0$, become comparably light. Such approximate $U(1)_a$ restoration is not however based on 'instanton suppression', but rather on new mechanism, a reorganization of the 'liquid' into into the instanton-anti-instanton

⁸ Unfortunately, it can only be done statistically, so 'fluctuations in eta direction' on event-per-event basis can only be done indirectly, via fluctuations in total number of photons.

molecules.

Finally, if these phenomena take place, large fluctuation of η production in heavy ion collision, especially at small transverse momenta, are predicted.

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